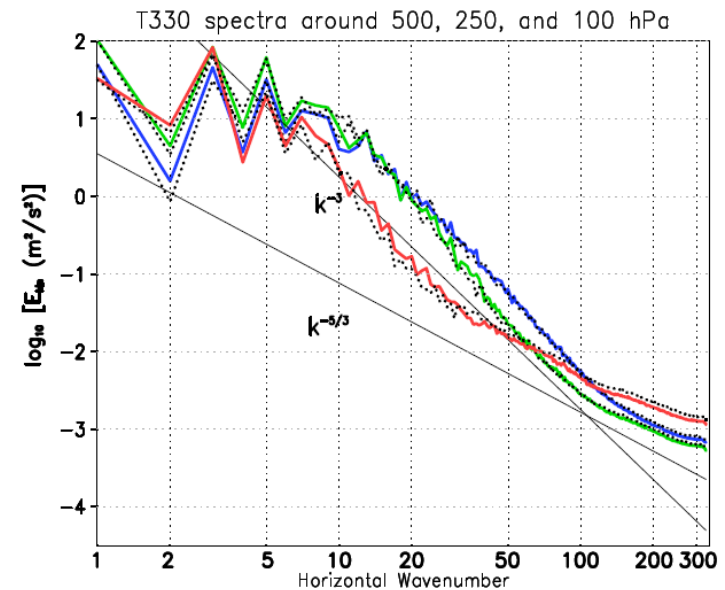
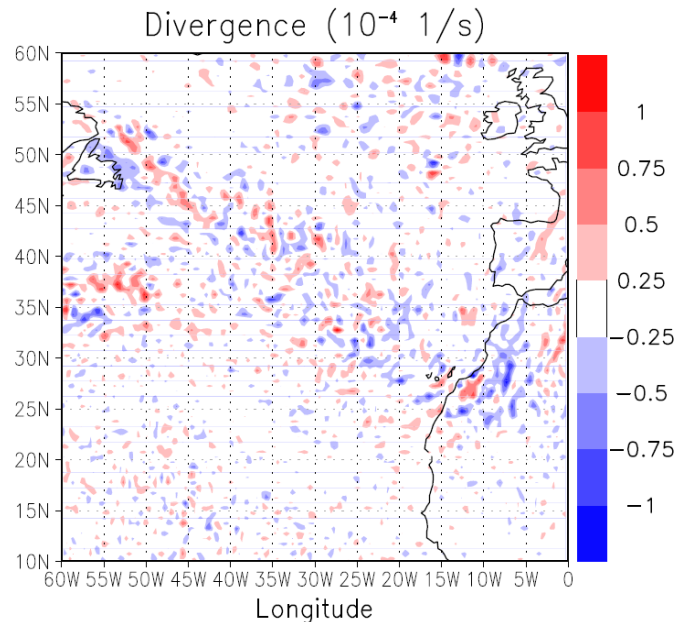


Application of a dynamic turbulence parametrization in a circulation model

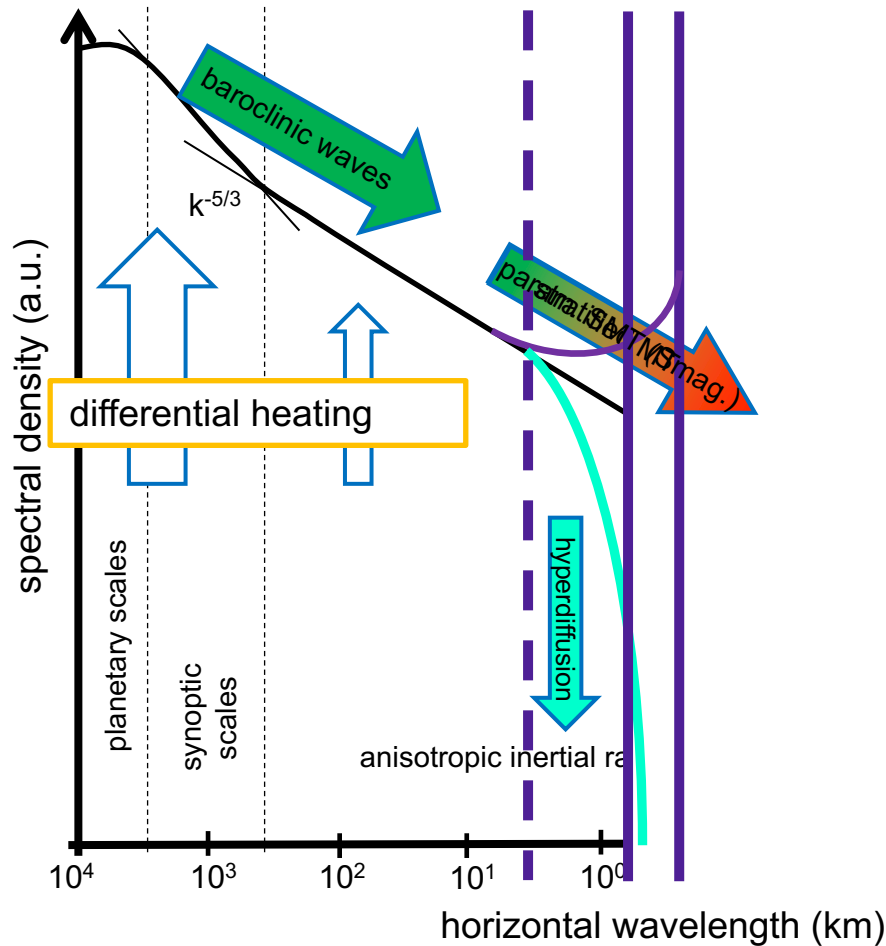
Outline:

- Retrospective: A consistent turbulence parametrization
- Stratified turbulence – a summary
- Results



Application of a dynamic turbulence parametrization in a circulation model

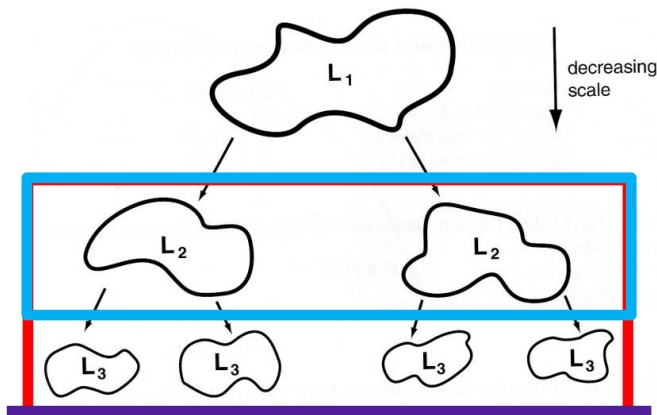
Retrospective: A consistent turbulence parametrization



Application of a dynamic turbulence parametrization in a circulation model

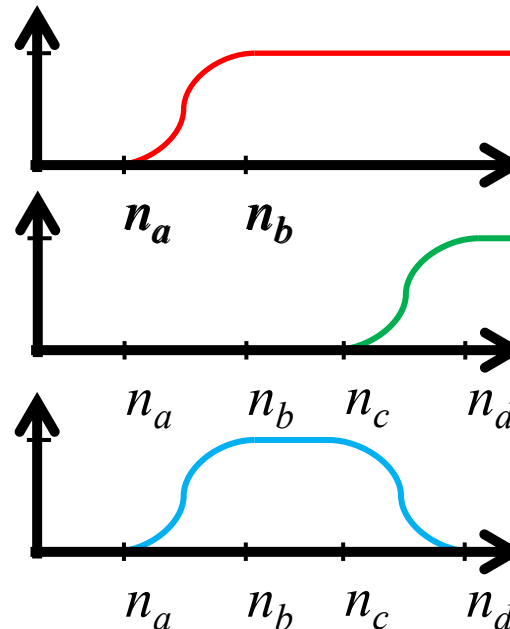
Retrospective: A consistent turbulence parametrization

- Mixing length approach for turbulent harmonic diffusion: $K_T = l_h^2 |\mathbf{S}_h|$
- Dynamic idea: Estimation of a dynamic mixing length $l_h = c_S \Delta_h$ by applying filter and assumption of self similarity
- Standard DSM: reasonable results, but depends on resolution scale, thus gDSM (Schaefer-Rolffs, Met. Z, 2017)



Scales below L_3 are unresolved scales. The test filter fades out scales of L_2 and L_3 (DSM, red), or only L_2 (gDSM, blue).

From: G. Vallis, Atmospheric and Oceanic Fluid Dynamics



Standard DSM: The test filter algorithm is applied once to the prognostic variables (top panel).

gDSM: The test filter algorithm is applied twice to the prognostic variables (top and central panel). The difference of these filtered variables results in a test filter regime (bottom panel) that is denoted hereafter $n_a/n_b // n_c/n_d$.

Application of a dynamic turbulence parametrization in a circulation model

Retrospective: A consistent turbulence parametrization

- Introduce scaling transformation of Euler Equations

$$t^* = e^{\frac{2}{3}c_x t}; \quad x_i^* = e^{c_x x_i}; \quad v_i^* = e^{\frac{1}{3}c_x v_i}; \quad a^* = e^{c_a a}; \quad b_l^* = e^{c_{b_l} b_l}$$

- Implications for stratified fluids: Parametrization of vertical diffusion in horizontal momentum equation

$$\rho^{-1} \partial_z (\rho K_z \partial_z \mathbf{u})$$

if the mixing length in $K_z = l_z^2 |\partial_z \mathbf{u}|$ obeys scale ratio $c_{l_z} = \frac{3c_z - c_x}{2}$

→ However, without knowing c_z , c_{l_z} is also not determined

Application of a dynamic turbulence parametrization in a circulation model

Retrospective: A consistent turbulence parametrization

New dynamic approach for l_z from scale-invariance criterion and stratified turbulence:

- Concept of Stratified Turbulence (Lindborg, J. Fluid Mech., 2006) shows aspect ratio, $Z/X \sim \varepsilon^{1/3} X^{-2/3} / N$
- With $N^2 = \text{const.}$, the ratio yields $c_z = \frac{1}{3} c_x$, which could be applied in two ways:
 1. Assuming that the ratio is valid for vertical scales, then $c_{l_z} = \frac{3c_z - c_x}{2} = 0$ and $l_z = \text{const.}$, in accordance to the classic Smagorinsky model
 2. Assuming that the ratio is valid for the mixing lengths, $c_{l_z} = \frac{1}{3} c_{l_h} = \frac{1}{3} c_x$ allowing to formulate a new dynamic vertical mixing length (DVML),

$$l_z \propto l_h^{1/3}$$

Application of a dynamic turbulence parametrization in a circulation model

Stratified turbulence – a summary

Two requirements: **Strong anisotropy** and **turbulence**

- **Anisotropy** of the atmosphere: Low horizontal Froude number $F_h = \frac{U}{NL_h} \ll 1$ (and $T = \frac{L_h}{U}$)
- In Boussinesq approximation: Non-dimensionalized equations reveal **vertical Froude number** $F_z = \frac{U}{NL_z}$ accompanying vertical velocity w (Billant and Chomaz, Phys. Fluids 2001)
- Scale analysis of Billant and Chomaz: Transformation group for z , w , N , and ρ with $c_z = c_w = -c_N = -c_\rho$; hence $L_z \propto \frac{1}{N}$
- Dimensional analysis yields $L_z \sim \frac{U}{N}$ or ~~$L_z \sim \frac{w}{N}$~~

$$\rightarrow F_z \sim 1$$

Application of a dynamic turbulence parametrization in a circulation model

Stratified turbulence – a summary

Two requirements: **Strong anisotropy** and **turbulence**

- **Turbulence analysis** (Taylor, Proc R. Soc. A 1935): $U \sim (\epsilon L)^{1/3}$
- Hence: $F_z = \frac{U}{NL_z} \sim 1$ and $U \sim (\epsilon L_h)^{1/3}$ for **stratified turbulence** (Lindborg, J. Fluid Mech. 2006)
- Aspect ratio: $\frac{L_z}{L_h} \sim \frac{\epsilon^{1/3}}{N L_h^{2/3}}$ or $L_z \sim \frac{\epsilon^{1/3}}{N} L_h^{1/3}$
- Assumption: Horizontal/vertical energy spectra $E_k \sim U^2 L$ depend only on horizontal/vertical length scale, thus
$$E_h \sim U^2 L_h \sim \epsilon^{2/3} L_h^{5/3} \sim \epsilon^{2/3} k_h^{-5/3}$$
$$E_z \sim U^2 L_z \sim N^2 L_z^3 \sim N^2 k_z^{-3}$$
- Further: Frequency spectrum exhibits $E_\omega \sim \omega^{-2}$ relation

Application of a dynamic turbulence parametrization in a circulation model

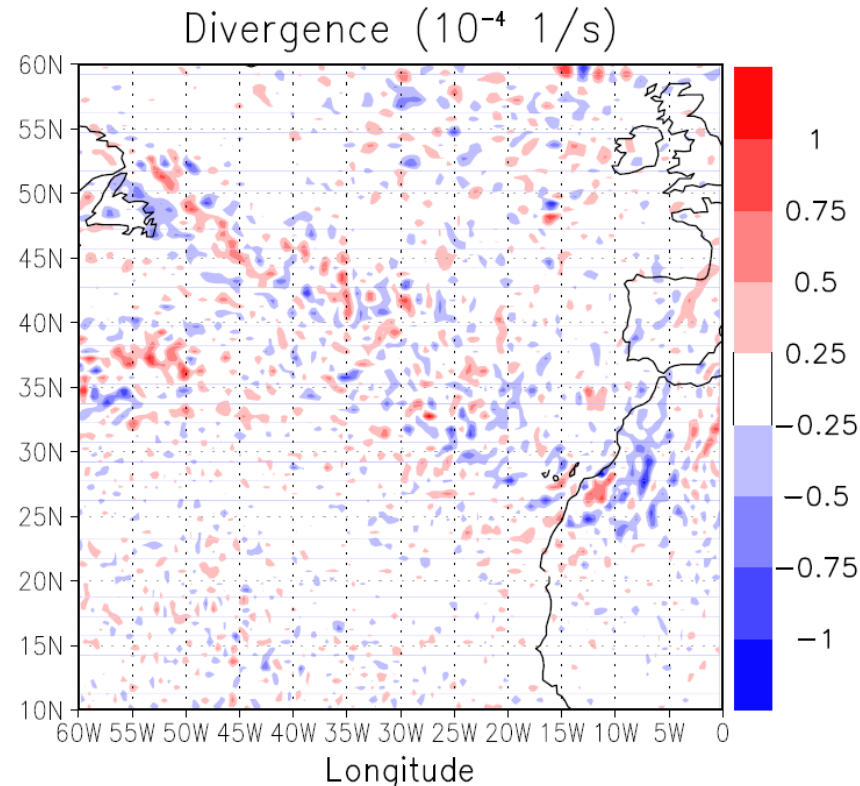
Stratified turbulence – a summary

Two requirements: **Strong anisotropy** and **turbulence**

- **Anisotropy** of the atmosphere: Low horizontal Froude number $F_h = \frac{U}{NL_h} \ll 1$ (and $T = \frac{L_h}{U}$)
- In Boussinesq approximation: Non-dimensionalized equations reveal **vertical Froude number** $F_z = \frac{U}{NL_z}$ (Billant and Chomaz, Phys. Fluids 2001)
- Magnitude of kinetic and potential energy
$$KE \sim (1 + F_h^2 F_z^2) U^2$$
$$PE \sim F_z^2 U^2$$
- Hence: Equipartition of kinetic and potential energy, similar as for gravity waves

Application of a dynamic turbulence parametrization in a circulation model

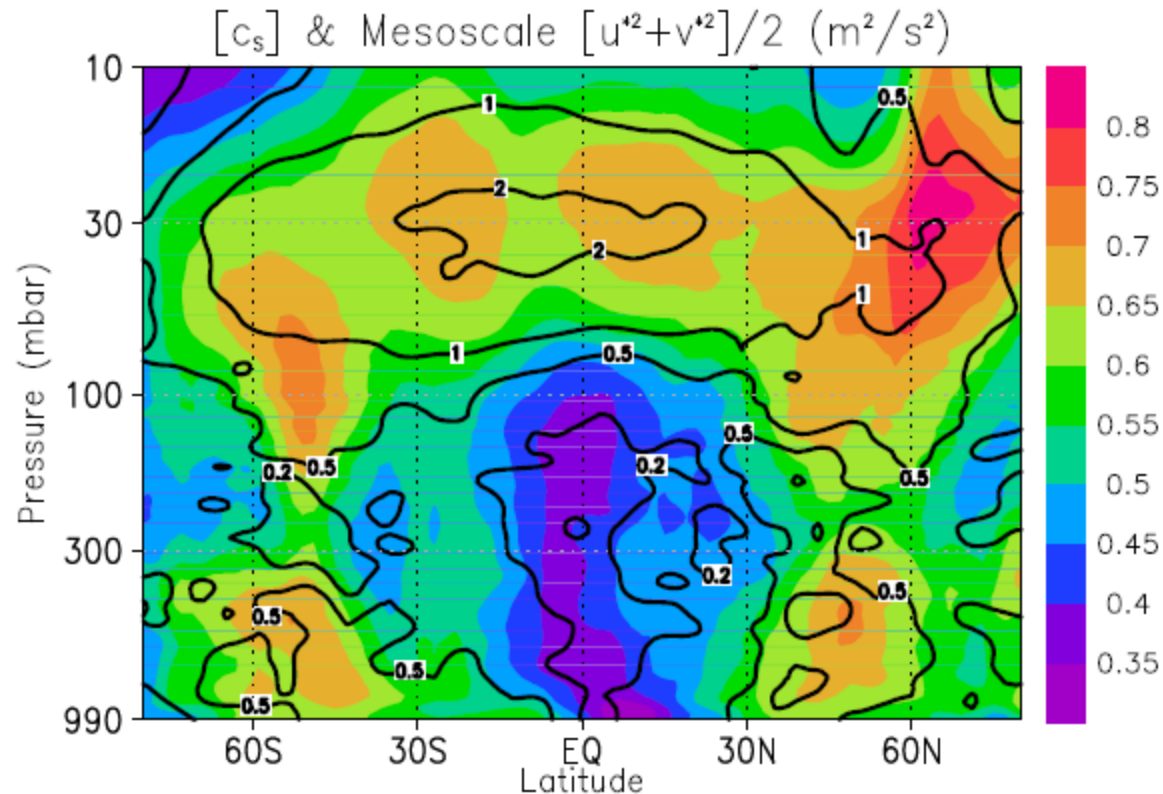
Results: Snapshot of horizontal divergence at 100 hPa



- The mesoscale activity is induced by the breakdown of baroclinic Rossby waves, reflecting the instantaneous gravity-wave activity

Application of a dynamic turbulence parametrization in a circulation model

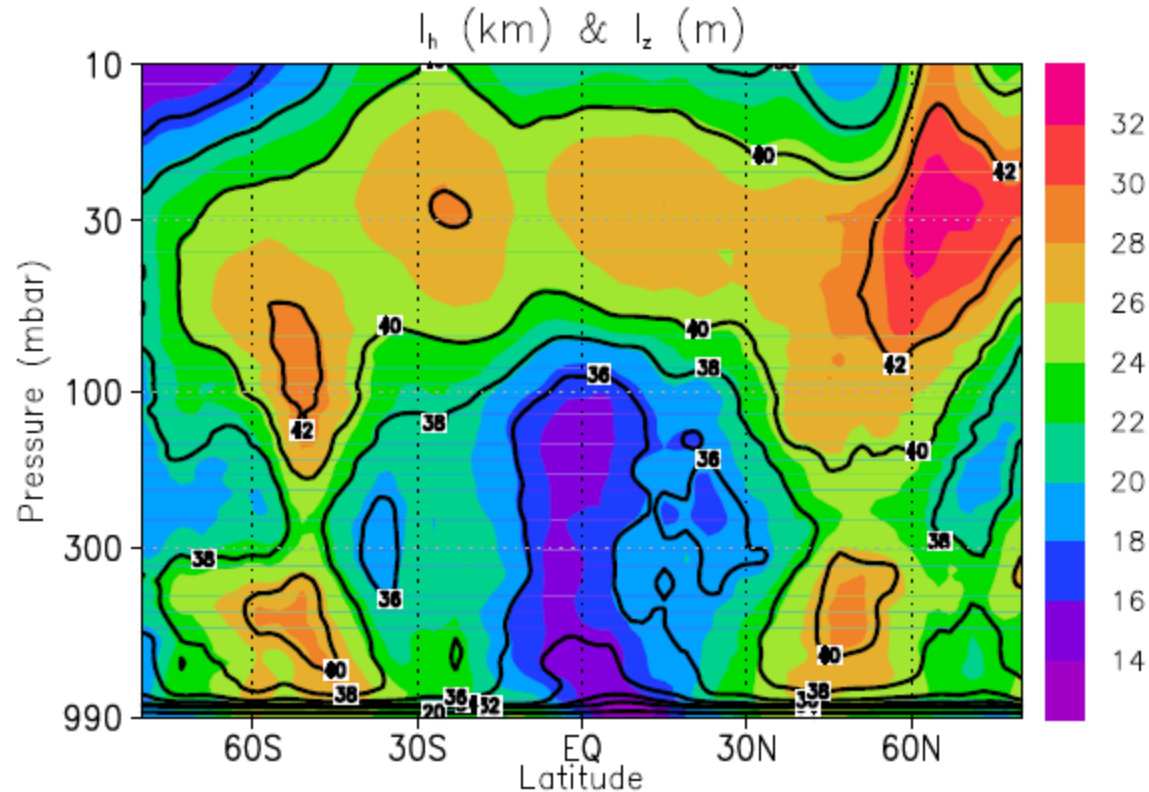
Results: Smagorinsky parameter (colors) and eddy kinetic energy (contours)



- The mesoscale eddy kinetic energy includes only wavenumbers $n > 90$

Application of a dynamic turbulence parametrization in a circulation model

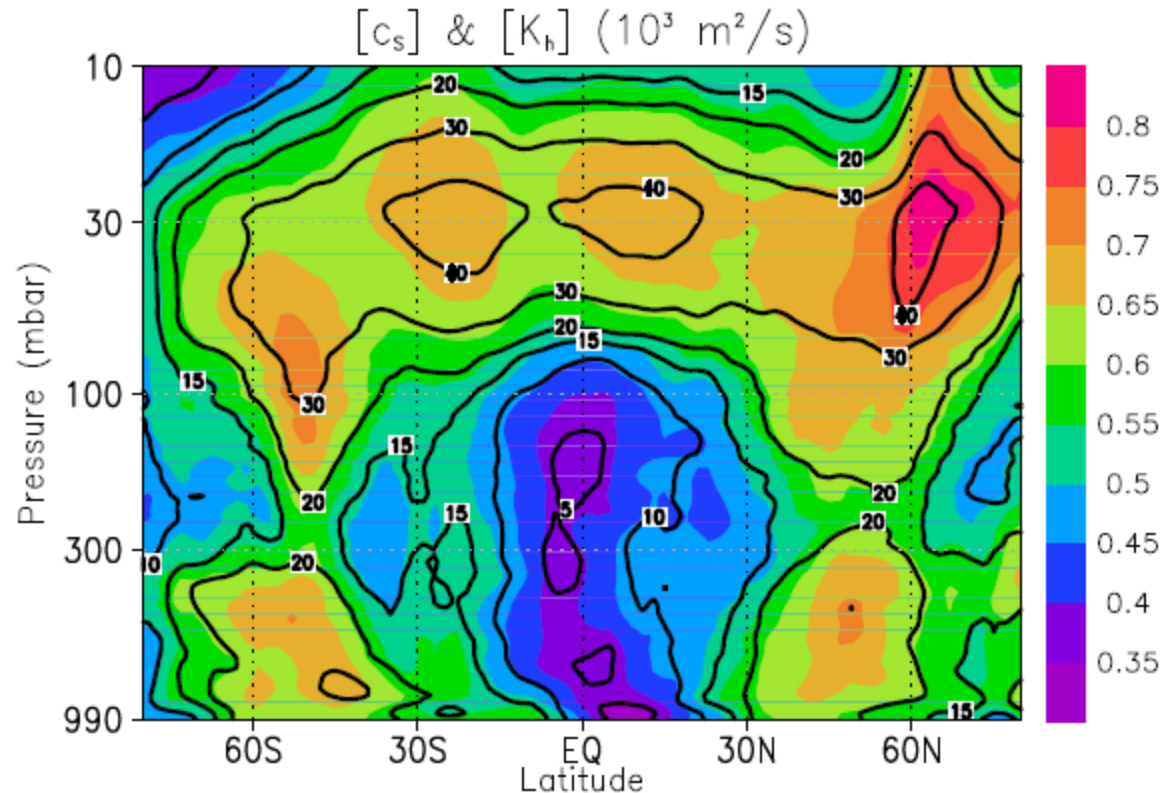
Results: Horizontal (colors) and vertical mixing lengths (contours)



- Due to $l_z \propto l_h^{1/3}$, the variations of l_z are much smaller than the variations of l_h

Application of a dynamic turbulence parametrization in a circulation model

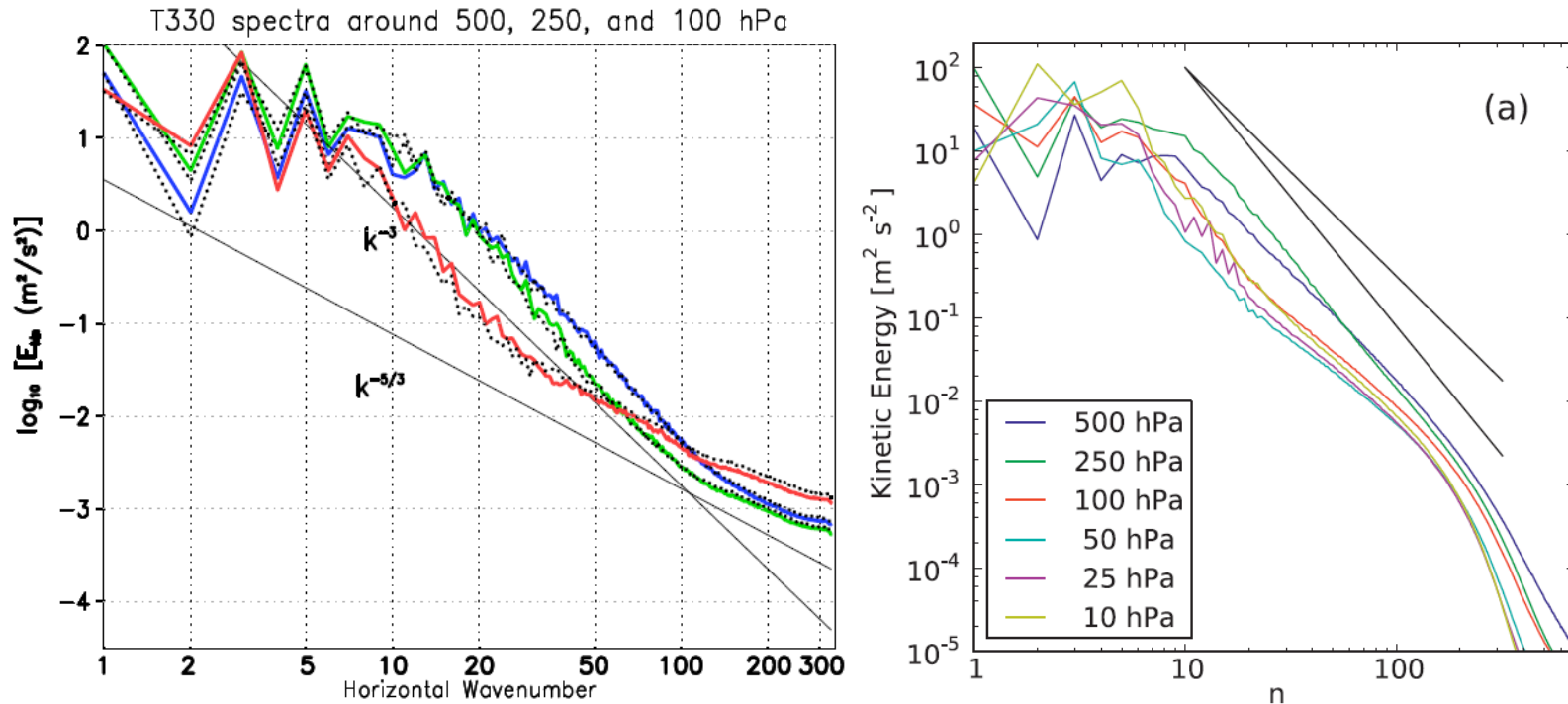
Results: Smagorinsky parameter (colors) and diffusion coefficient (contours)



- The variability of $K_h = l_h^2 |S_h|$ is controlled by the variability of c_s , unlike to the CSM

Application of a dynamic turbulence parametrization in a circulation model

Results: Kinetic energy spectra at 500 (blue), 250 (green), and 100 (red) hPa

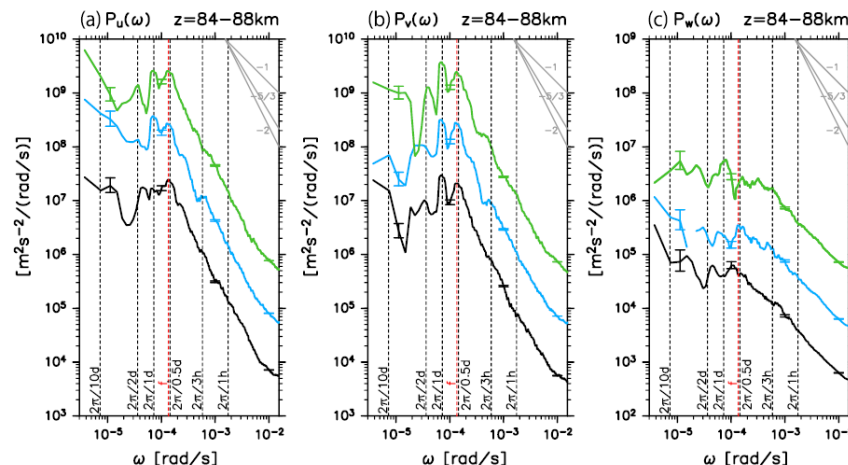


- The black dotted curves are the energy spectra that are obtained when using a constant asymptotic vertical mixing length of 30 m
- Similar transition as seen in ECMWF analyses (Burgess et al, J. Atmos. Sci., 2013)

Application of a dynamic turbulence parametrization in a circulation model

Region of interest for IAP: mesosphere/lower thermosphere (MLT, 60...110km)

- Developments and changes in MLT **much stronger** than in troposphere
- (Secondary) gravity waves have **higher wavelengths** in MLT than in troposphere
- Assumption: **Stochastic generation** and **superposition** of (secondary) gravity waves in MLT is visible as stratified turbulence
- Experimental radio data (Sato et al, J. Geophys. Res. Atmos. 2017): $E_\omega \sim \omega^{-2}$ spectrum at 84...88 km



Application of a dynamic turbulence parametrization in a circulation model

Outlook:

- Investigation of GW spectra in the MLT with respect to scaling ratio of stratified macro-turbulence
 - **Framework is set** (Schaefer-Rolffs and Becker, Mon. Wea. Rev. 2018), **extension to MLT will follow**
- Comparison with GW measurements in MLT: new lidar/radar techniques detect GW in lower thermosphere with high temporal and horizontal resolutions
 - **Experimental set-up of MMARIA (Multistatic/Multifrequency Agile Radar for Investigations of the Atmosphere) started in 2014, will be extended**
 - **Approval for VAHCOLI (Vertical And Horizontal COverage by LIdar) in January 2018**
- Application of gDSM to calculate turbulent diffusion of
 - temperature
 - tracers
 - self-consistent Prandtl/Schmidt numbers
 - **Two Postdocs within the TRR 181 „Energy transfers in Atmosphere and Ocean“**



