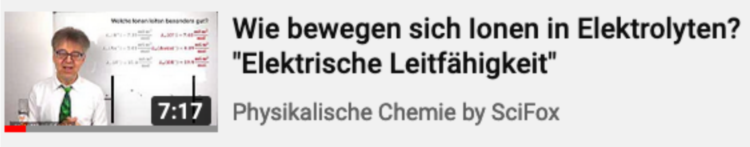
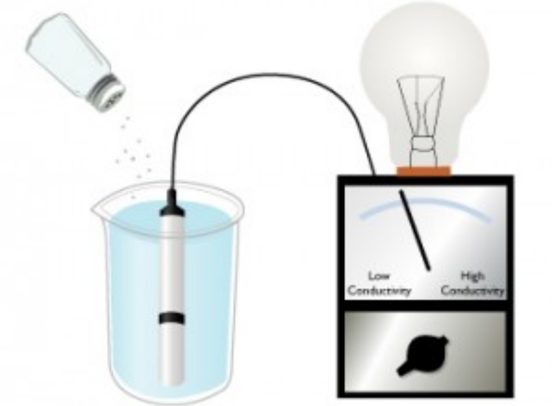
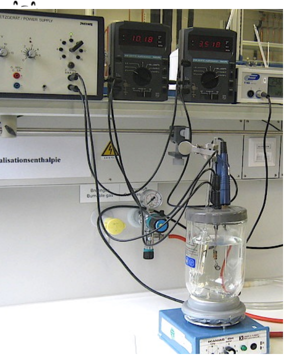


PHYSICAL CHEMISTRY BASICS
PART 11: ELECTROLYTIC CONDUCTIVITY

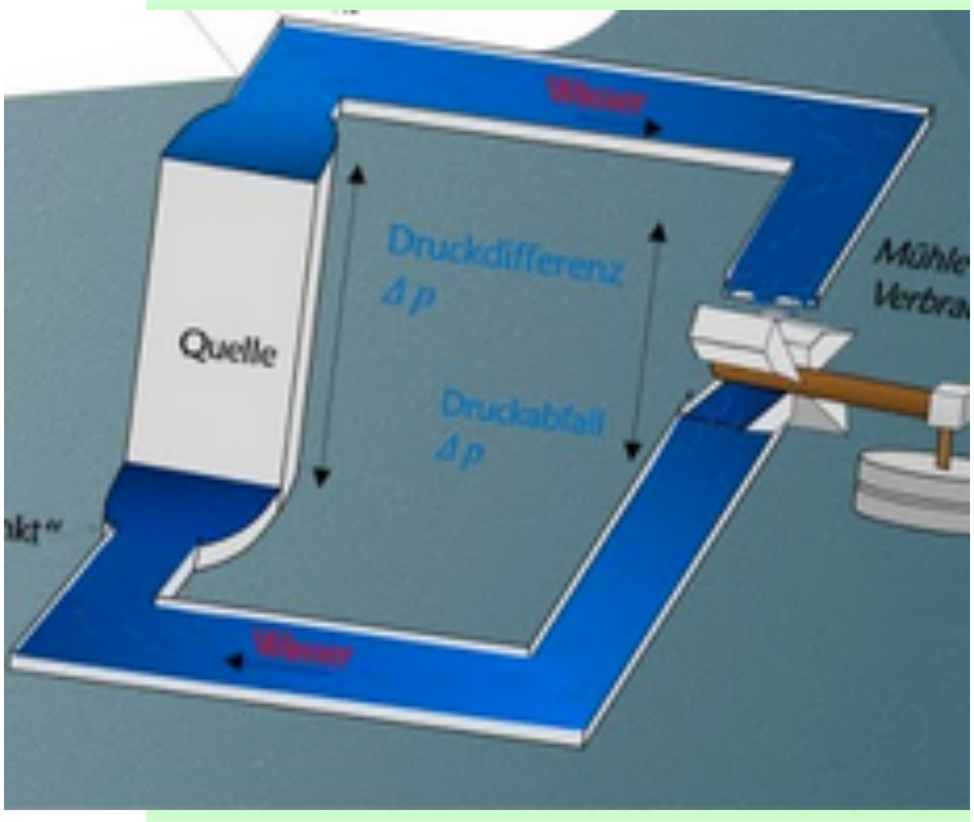
How does current flow through an electrolyte?



$$\kappa = ?$$

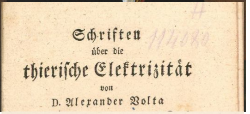


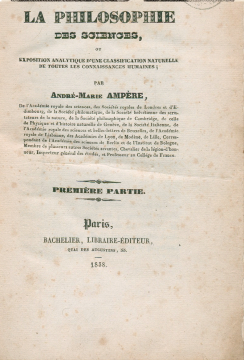
Electricity Basics I

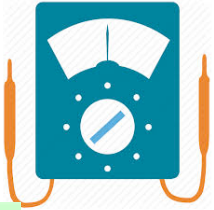


#Charge #Circuit #Polarity

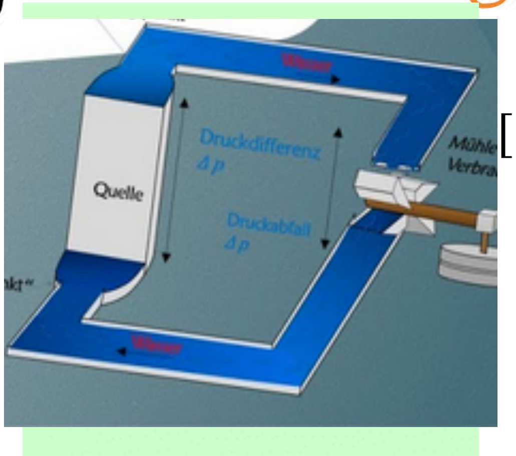
Electricity Basics II


Schriften
über die
thierische Elektrizität
von
D. Alexander Volta


LA PHILOSOPHIE
DES SCIENCES,
OU
EXPOSITION ANALYTIQUE D'UNE CLASSIFICATION NATURELLE
DE TOUTES LES CONNAISSANCES HUMAINES;
PAR
ANDRÉ-MARIE AMPÈRE,
PREMIÈRE PARTIE.
Paris,
BACHELIER, LIBRAIRE-ÉDITEUR,
1838.



$[I] = A$

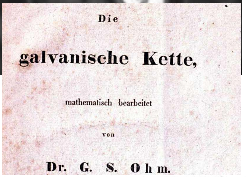


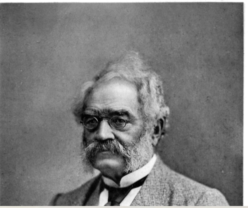
$[U] = \mathrm{V}$

#Voltage

#Current

Electricity Basics III


Die
galvanische Kette,
mathematisch bearbeitet
von
Dr. G. S. Ohm.

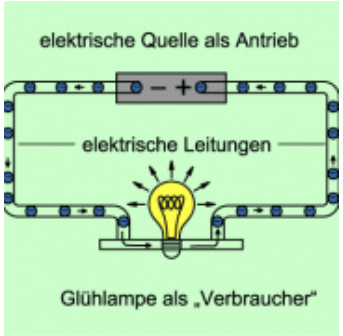



Lebenserinnerungen
von
Werner von Siemens.

#Resistance

#Conductance

#Ohm'sLaw



$[U] = \mathrm{V}$

$$\boldsymbol{R = \frac{U}{I}}$$

$[R] = \Omega$

$$\boldsymbol{G = \frac{1}{R} = \frac{I}{U}}$$

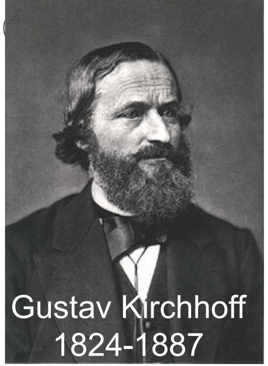
$[G] = S$

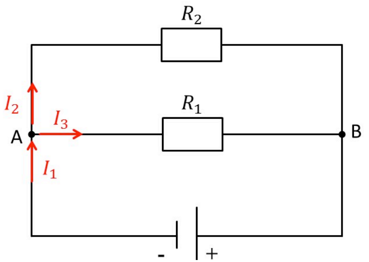
$[I] = A$

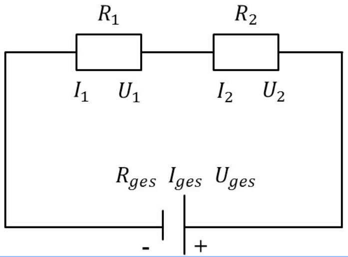
$$\boldsymbol{P = U \cdot I}$$

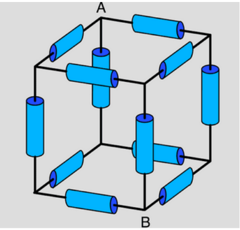
$[P] = W$

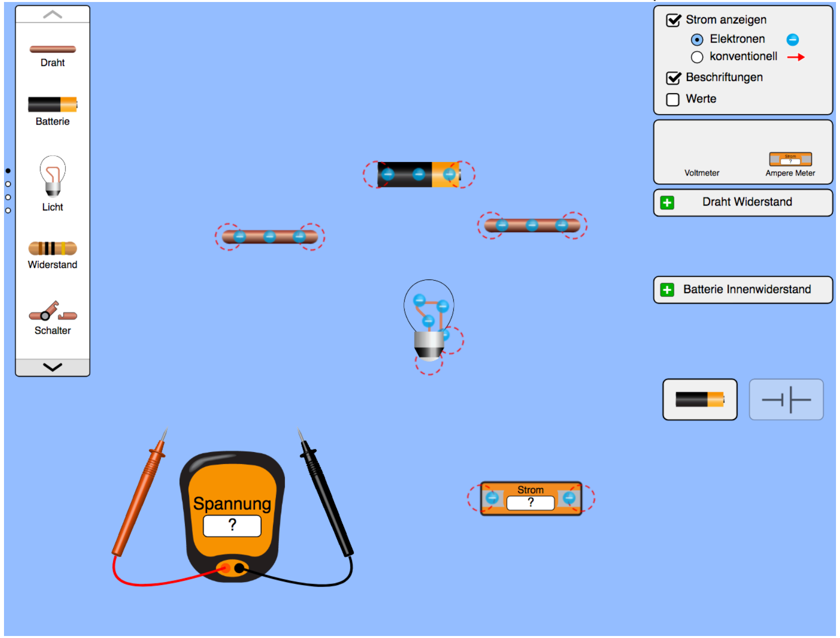
Electricity Basics IV











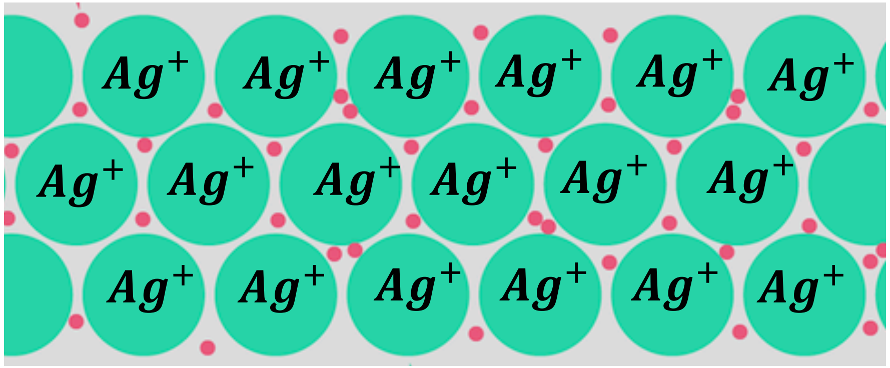
#ParallelConnection

#SeriesConnection

#Kirchhoff'sLaws

übersetzt von: Prof. Dr. Jakob "SciFox" Lauth, University of Applied Sciences, Jülich Campus

What are the **charge carriers** when electricity flows through a metal?



$\circ = e^-$

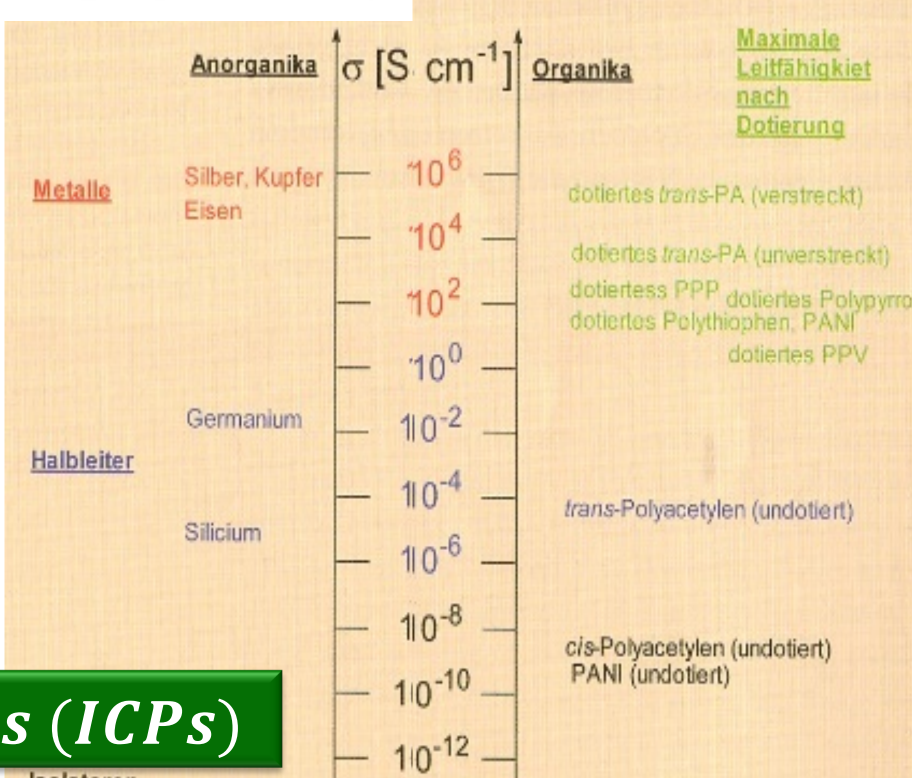
$$Q = -e = \mathbf{1.6 \cdot 10^{-19}\ As} \qquad (V-2.1)$$

$$e \cdot N_A = \mathbf{96\,485\,\frac{As}{mol}} = F$$

Polyacetylene Polythiophene Poly(p-phenylene)

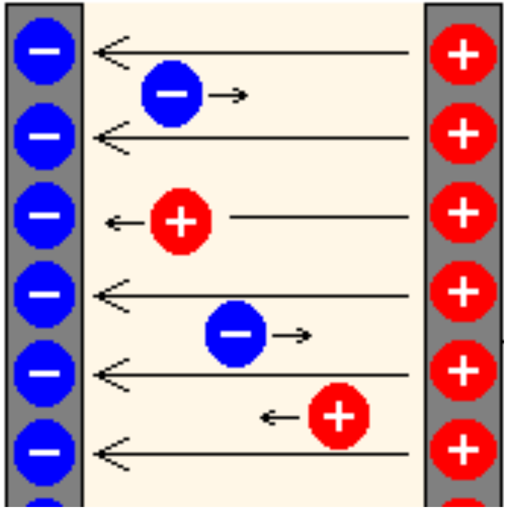
#ElectronicConductor

Are all electronic conductors **metals**?



#IntrisicConductingPolymers (ICPs)

What are the **charge carriers** when electricity flows through an electrolyte?



$$Q^+ = z^+ e \qquad (V-2.3)$$

$$Q^- = z^- e$$

$$K_{\nu^+}A_{\nu^-} \xrightarrow{\alpha} \nu^+ K^{z^+} + \nu^- A^{z^-} \qquad (V-2.2)$$

$$n_e = \nu^+ z^+ = |\nu^- z^-| \qquad (V-2.4)$$

$$\mathrm{NaCl} \rightarrow Na^+ + Cl^-$$

Elektrochemische Wertigkeit und van't Hoff'scher Faktor von Kupferchlorid

Kupferchlorid dissoziiert vollständig (α =1) in Kupferionen und Chloridionen:

$$CuCl_2 \rightarrow Cu^{2+} + 2Cl^- \qquad (2.5)$$

Aus 1 mol Kupferchlorid entstehen 2 mol positive und 2 mol negative Ladungen:

$$n_e = \nu_+ z_+ = \nu_- |z_-| = 2 \qquad (2.6)$$

Aus 1 mol Kupferchlorid entstehen aber 3 mol gelöste Teilchen:

$$i = (\nu_+ + \nu_- - 1)\alpha + 1 = 3$$

#ElectricalEquivalent

#ElectrolyticConductor

True or False?

A: In liquid mercury the charge carriers are ions

B: In Polyaniline the charge carriers are electrons

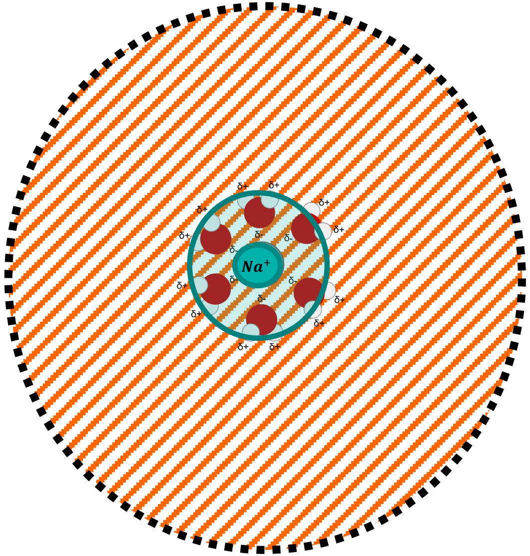
C: In pure solid silicon the charge carriers are electrons

D: In solid zirconia the charge carriers are ions

Peter Debye
1884 -1966

Erich Hückel
1896 - 1980

What is the **structure** of an electrolyte solution?



$$a_{\pm} = f_{\pm} \cdot c_{\pm}$$

(V – 3.8)

$$I = \frac{1}{2}\sum_i z_i^2 \cdot c_i$$

(V – 3.2)

$$\log f_{\pm} = 0.509\, z_+\, z_- \sqrt{\frac{I}{\text{mol/L}}}$$

(V – 3.13)

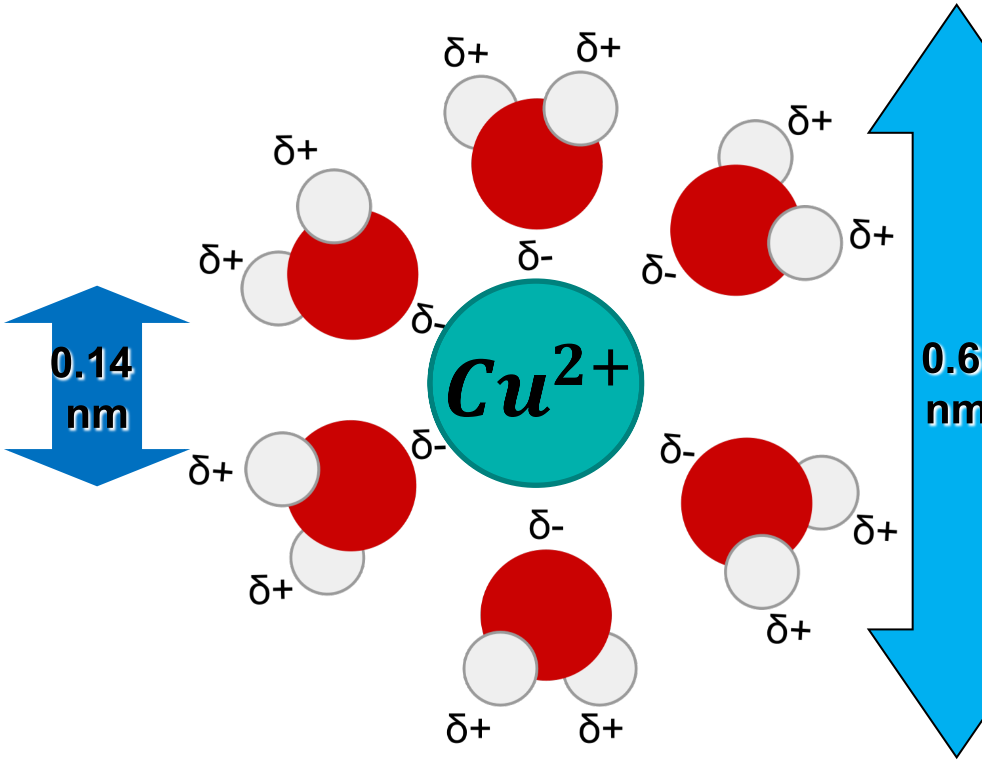
$I = c = 0.5\,\frac{\text{mol}}{\text{L}}$ $f_{\pm} = 0.7$ $a_{\pm} = 0.35\,\frac{\text{mol}}{\text{L}}$

#ActivityCoefficient

#IonicStrength

#DebyeHückelTheory

What are the **charge carriers** in electrolytic conductors?



#StokesRadius

#EffectiveIonicRadius

How do we quantify **ion concentration** in an electrolyte?

$[CuCl_2] = 0.001\,\frac{mol}{L}$

$$I = \frac{1}{2}\sum_i z_i^2 \cdot c_i \qquad (V-3.2)$$

$$CuCl_2 \xrightarrow{\alpha} Cu^{2+} + 2\,Cl^- \qquad (V-3.1)$$

$$I = \frac{1}{2}\left(z_{Cu^{2+}}^2 \cdot c_{Cu^{2+}} + z_{Cl^-}^2 \cdot c_{Cl^-}\right)$$

$$I = \frac{1}{2}\left(2^2 \cdot 1\,\frac{mmol}{L} + (-1)^2 \cdot 2\,\frac{mmol}{L}\right)$$

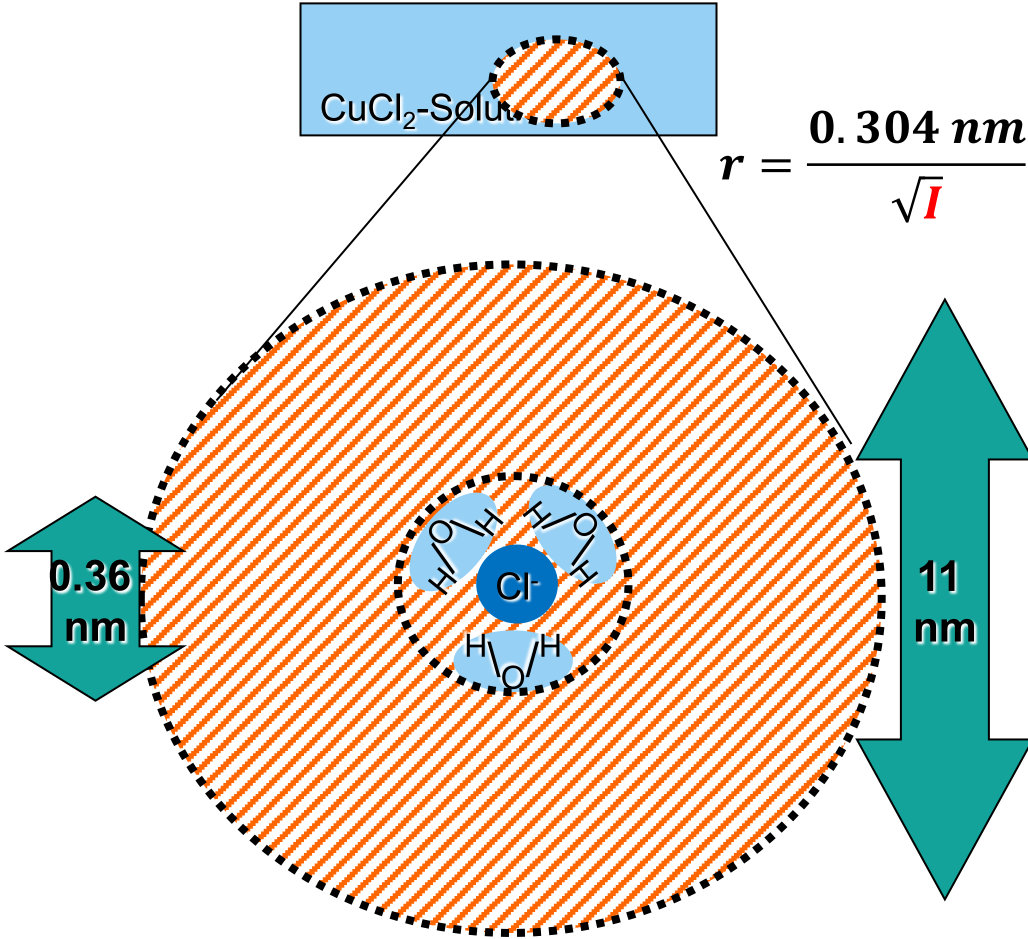
$$I = 0.003\,\frac{mol}{L} \qquad (V-3.5)$$

#IonicStrength

What is the effect of the **ionic cloud** around the central ion?

Peter Debye
1884 -1966

Erich Hückel
1896 - 1980



$$r = \frac{0.304\ nm}{\sqrt{I}}$$

#IonicAtmosphere

What is the **effective concentration** of a 0.001 M copper chloride solution?

$$\log f_{\pm} = 0.509\, z_{+}\, z_{-} \sqrt{\frac{I}{\text{mol/L}}}$$

(V − 3.13)

$$\log f_{\pm} = -0.509 \cdot |2 \cdot (-1)| \cdot \sqrt{0.003}$$

$$f_{\pm} = 10^{-0.0558}$$

$$f_{\pm} = 0.88$$

(V − 3.15)

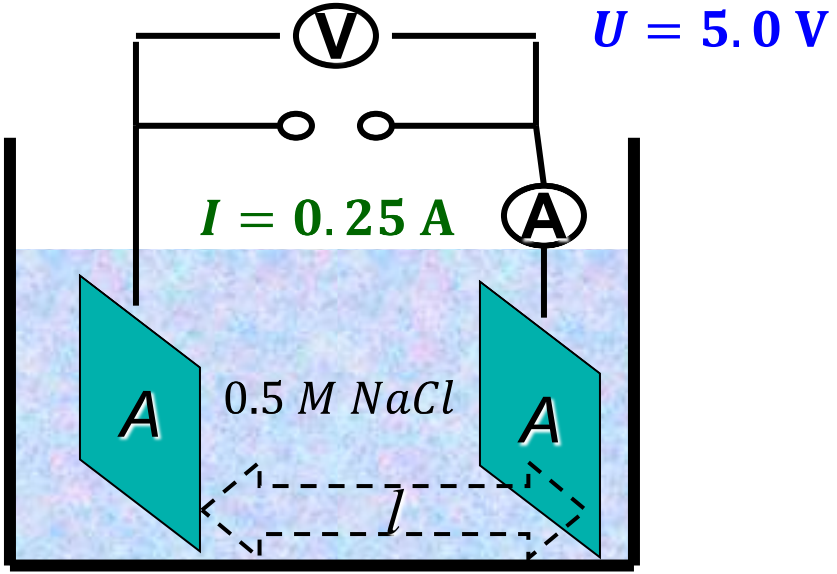
Tab. 3.1 Aktivitätskoeffizienten für 1-1-Elektrolyte

c / mol/L	I / mol/L	Theorie	HCl
0,001	0,001	0,9636	0,9656
0,002	0,002	0,9489	0,9521

#ActivityCoefficient

How do we measure the conductivity of an electrolyte solution?

$\kappa(H_2O) = 5.5\,\frac{\mu S}{m}$



$$R = \frac{U}{I} = \frac{5.0\ V}{0.25\ A} = 20\ \Omega \qquad (V-4.1)$$

$$\kappa = \frac{1}{R}\cdot\frac{l}{A} = \frac{1}{20\ \Omega}\cdot\frac{1\ \mathrm{m}}{0.01\ \mathrm{m}^2} = 5.0\,\frac{S}{m}$$

$(V - 4.4)$

#SpecificConductance

#Conductivity

Comparing the **conductivities** of different materials

Tab. 4.1 Spezifische elektrische Leitfähigkeit einiger Medien

Medium	*T*	
Cu (s)	25 °C	580.000 S/cm
Fe (s)	25 °C	96.100 S/cm

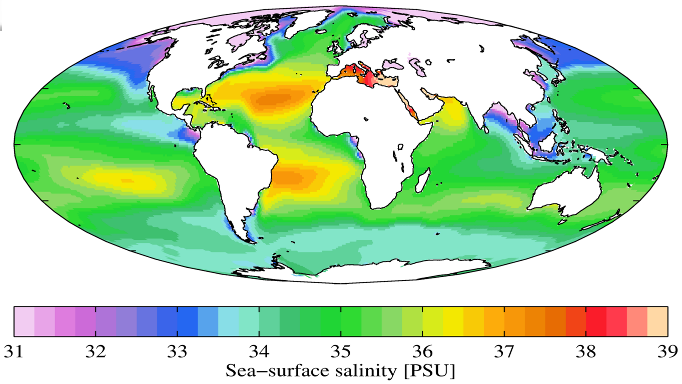
	0,01 M	0,1 M	1 M
18 °C	1,225 mS/cm	11,190 mS/cm	98,240 mS/cm
19 °C	1,251 mS/cm	11,430 mS/cm	100,160 mS/cm
20 °C	1,278 mS/cm	11,870 mS/cm	102,090 mS/cm
21 °C	1,305 mS/cm	11,910 mS/cm	104,020 mS/cm
22 °C	1,332 mS/cm	12,150 mS/cm	105,940 mS/cm
23 °C	1,359 mS/cm	12,390 mS/cm	107,890 mS/cm
24 °C	1,386 mS/cm	12,640 mS/cm	109,840 mS/cm
25 °C	1,413 mS/cm	12,880 mS/cm	111,800 mS/cm

#Conductometry

What is the conductivity of **water**?

$$H_2O \rightleftharpoons H^+ + OH^-$$

Water Type	Conductivity in $\frac{mS}{m}$
totally pure water	0.0055
typical deionized water	0.01
distilled water	$0.05 - 0.3$
reverse osmosis water	$5 - 10$
domestic "tap" water	$50 - 80$
potable water	$max\ 100$
sea water	≈ 5600



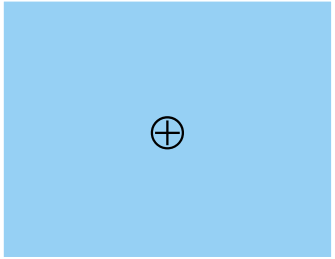
How does **concentration** affect conductivity?

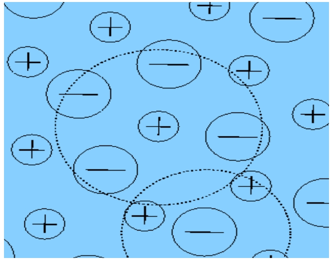
$$\kappa(H_2O) = 5.5\ \frac{\mu S}{m}$$

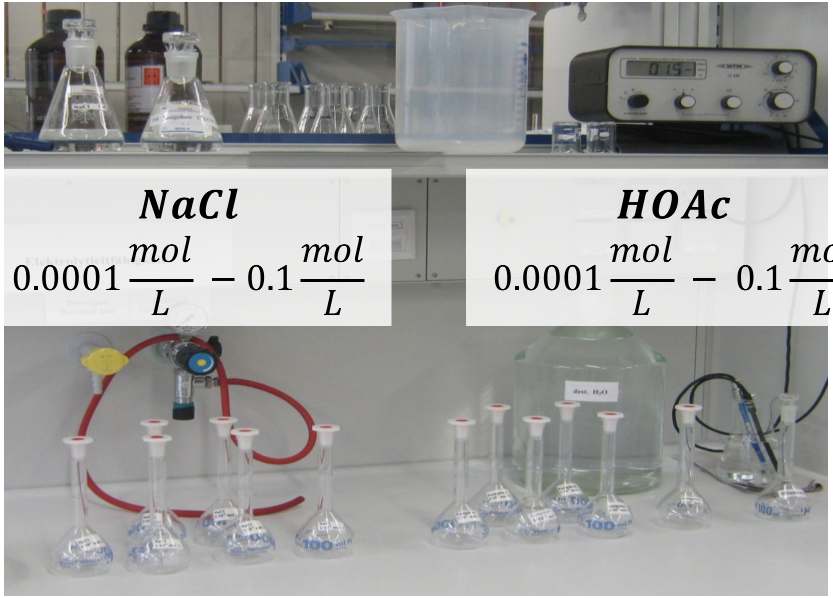
$$\kappa(Cu) = 58\ \frac{MS}{m}$$

$$\kappa(0.5\ M\ NaCl) = 5.0\ \frac{S}{m}$$

$$\kappa(0.05\ M\ NaCl) = 0.6\ \frac{S}{m}$$







#IonConcentration

#IonicMobility

How do we calculate the **molar conductivity** of a solution?

$$\kappa(0.5\ M\ NaCl) = 5.0\ \frac{\mathrm{S}}{\mathrm{m}}$$

$$\kappa(0.05\ M\ NaCl) = 0.6\ \frac{\mathrm{S}}{\mathrm{m}}$$

$$\kappa = \frac{1}{R}\frac{l}{A}$$

$$\Lambda_m = \frac{\kappa}{c}$$

$(V - 4.5)$

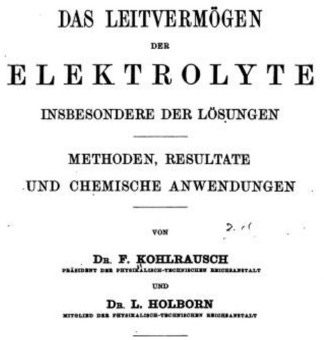
$$\Lambda = \frac{\kappa}{c \cdot n_e}$$

$$\Lambda = \frac{5.0\,\frac{\mathrm{S}}{\mathrm{m}}}{500\,\frac{\mathrm{mol}}{\mathrm{m}^3} \cdot 1} = 10\ \frac{\mathrm{mS\ m^2}}{\mathrm{mol}}$$

#MolarConductivity

#EquivalentConductivity

How does molar conductance vary with **dilution**?



DAS LEITVERMÖGEN
DER
ELEKTROLYTE
INSBESONDERE DER LÖSUNGEN

METHODEN, RESULTATE
UND CHEMISCHE ANWENDUNGEN

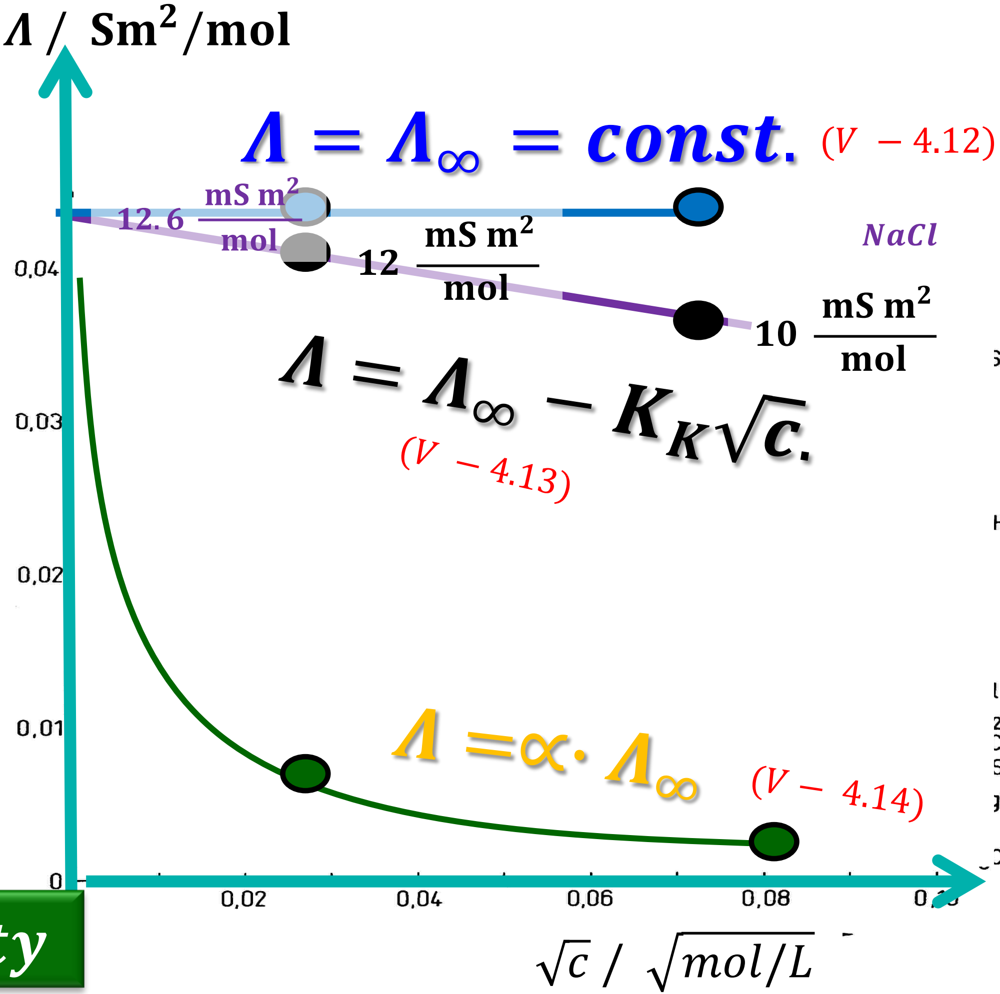
VON

DR. F. KOHLRAUSCH
PRÄSIDENT DER PHYSIKALISCH-TECHNISCHEN REICHSANSTALT
UND
DR. L. HOLBORN
MITGLIED DER PHYSIKALISCH-TECHNISCHEN REICHSANSTALT

$$\Lambda_\infty(KCl) = 14.98\frac{mS\,m^2}{mol}$$

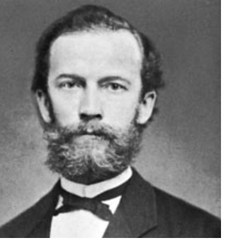
$$\Lambda_\infty(NaCl) = 12.64\frac{mS\,m^2}{mol}$$

$$\Lambda_\infty(CH_3COOH) = 39.07\frac{mS\,m^2}{mol}$$



#LimitingMolarConductivity

How do we calculate **limiting molar conductivity (l)**?



Friedrich Kohlrausch
1840 - 1910

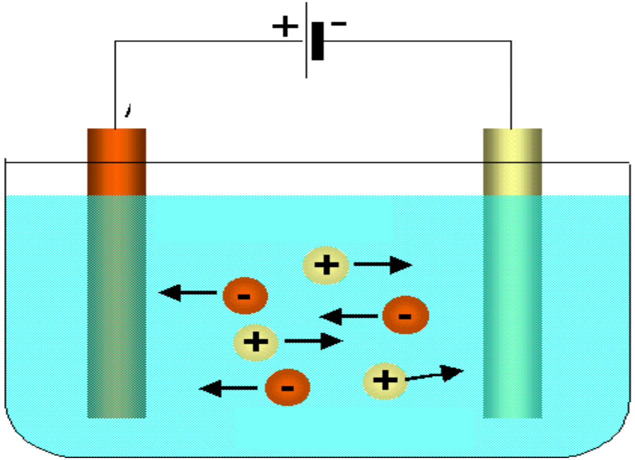
$\lambda_\infty \ in \ \frac{\text{mS m}^2}{\text{mol}}$

$Na^+: \ 5.01$

$Cl^-: 7.63$

$H^+: \ 34.98$

$AcO^-: 4.09$

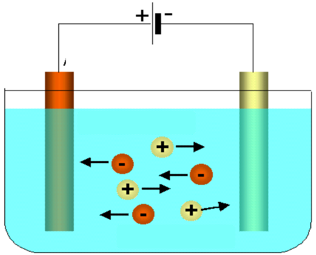


$$\Lambda_\infty = \lambda_{+\infty} + \lambda_{-\infty}$$

$(V - \ 4.15)$

#IonicConductivity

#IndependentIonicMigration



How do we calculate **limiting molar conductivity (II)**?

$$\Lambda_\infty = \lambda_{+\infty} + \lambda_{-\infty}$$

(V − 4.15)

λ_∞ *in* $\frac{\text{mS m}^2}{\text{mol}}$

H^+: 34.98

AcO^-: 4.09

K^+: 7.35

Cl^-: 7.63

Na^+: 5.01

$$KCl \xrightarrow{\alpha} K^+ + Cl^-$$

(V − 4.18)

$$\Lambda_\infty = 7.35 + 7.63 = 14.98 \frac{mS\, m^2}{mol}$$

$$NaCl \xrightarrow{\alpha} Na^+ + Cl^-$$

$$\Lambda_\infty = 5.01 + 7.63 = 12.64 \frac{mS\, m^2}{mol}$$

$$H_3CCOOH \xrightarrow{\alpha} H^+ + H_3CCOO^-$$

$$\Lambda_\infty = 34.98 + 4.09 = 39.07 \frac{mS\, m^2}{mol}$$

#IonicConductivity

#IndependentIonicMigration

How to cations and anions **contribute** to conductivity?

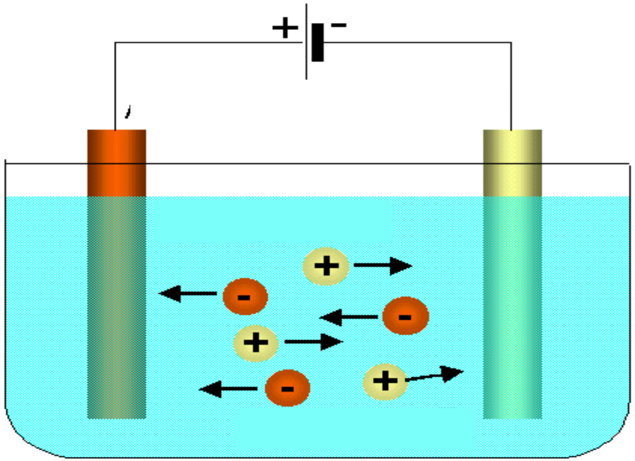
λ_∞ *in* $\frac{\mathrm{mS\ m^2}}{\mathrm{mol}}$

Na^+: 5.01

Cl^-: 7.63

H^+: 34.98

AcO^-: 4.09

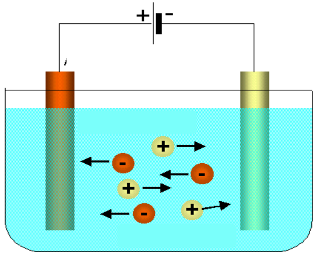


$$t_+ = \frac{\lambda_{+\infty}}{\Lambda_\infty} \qquad t_- = \frac{\lambda_{-\infty}}{\Lambda_\infty}$$

$(V - 4.16)$

#TransportNumber

How do we calculate **transfer numbers**?



$\lambda_\infty \; in \; \frac{\text{mS m}^2}{\text{mol}}$

$H^+: \; 34.98$

$AcO^-: 4.09$

$K^+: \; 7.35$

$Cl^-: 7.63$

$Na^+: \; 5.01$

$$t_+ = \frac{\lambda_{+\infty}}{\Lambda_\infty}$$

$$KCl \xrightarrow{\alpha} K^+ + Cl^-$$

$$t_+ = \frac{7.35}{14.98} = 0.49 \quad (V - 4.19)$$

$$\text{Na}Cl \xrightarrow{\alpha} Na^+ + Cl^-$$

$$t_+ = \frac{5.01}{12.64} = 0.40$$

$$H_3CCOOH \xrightarrow{\alpha} H^+ + H_3CCOO^-$$

$$t_+ = \frac{34.96}{39.07} = 0.90$$

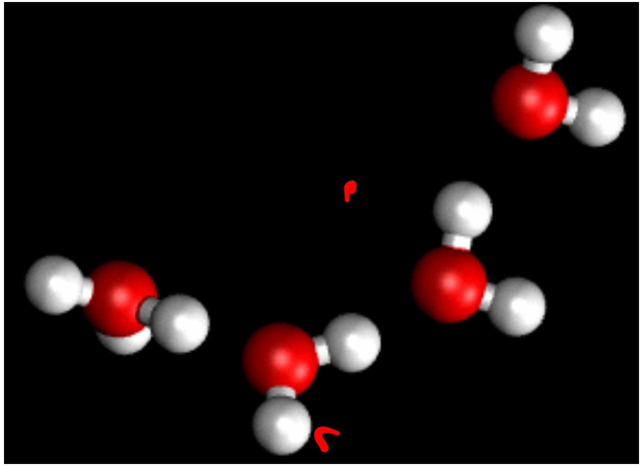
What is the effect of **ionic radius** on conductivity?

$$\lambda_\infty(K^+) = 7.35\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}} \qquad \lambda_\infty(Cl^-) = 7.63\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}}$$

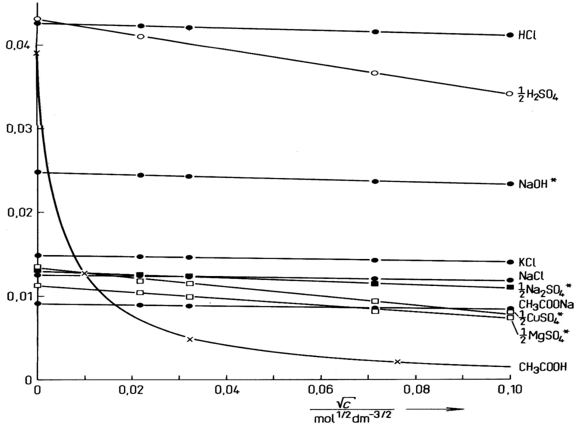
$$\lambda_\infty(Na^+) = 5.01\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}} \qquad \lambda_\infty(Acetate^-) = 4.09\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}}$$

$$\lambda_\infty(H^+) = 35.0\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}} \qquad \lambda_\infty(OH^-) = 19.9\,\frac{\mathrm{mS\,m^2}}{\mathrm{mol}}$$

$(V - Tab. 4.2)$



#GrotthußMechanism



True or False?

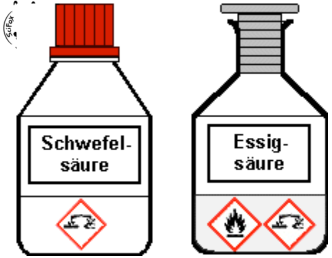
Consider a concentrated and a dilute solution of hydrochloric acid (HCl)

A: The specific conductivities of the solutions are the same.

B: The molar conductivities of the two solutions are the same.

C: The equivalent conductivities of the solutions are the same.

D: The limiting conductivities of the two solutions are the same.



Calculation conductivity of a **weak and a strong electrolyte**

2. Berechnen Sie die spezifische Leitfähigkeiten κ_E sowie die pH-Werte ($pH = -\log\{c_{H+}\}$) folgender Säurelösungen bei 25 °C:

- 1 mmol Schwefelsäure (idealer Elektrolyt) in 1 L Wasser.
- 1 mmol Essigsäure ($H_3C\text{-}COOH$) in 1 L Wasser.
- In beiden Fällen kann die Wirkung der Ionenwolke vernachlässigt werden.

$(V - 11.14 \;\; - 11.35)$

PCÜ43 Leitfähigkeit starker & schwacher Elektrolyte - wie gut leiten Schwefelsäure & Essigsäure?

Physikalische Chemie by SciFox

Table of limiting ion conductivity in water at 298K (approx. 25°C)[9]

Cations	λ_+^o / mS·m²·mol⁻¹	Cations	λ_+^o / mS·m²·mol⁻¹	Anions	λ_-^o / mS·m²·mol⁻¹	Anions	λ_-^o / r
H^+	34.982	Ba^{2+}	12.728	^-OH	19.8	SO_4^{2-}	15.96
Li^+	3.869	Mg^{2+}	10.612	Cl^-	7.634	$C_2O_4^{2-}$	7.4
Na^+	5.011	La^{3+}	20.88	Br^-	7.84	$HC_2O_4^{1-}$	40.2
K^+	7.352	Rb^+	7.64	I^-	7.68	$HCOO^-$	5.6
NH_4^+	7.34	Cs^+	7.68	NO_3^-	7.144	CO_3^{2-}	7.2
Ag^+	6.192	Be^{2+}	4.50	CH_3COO^-	4.09	HSO_3^{2-}	5.0
Ca^{2+}	11.90			ClO_4^-	6.80	SO_3^{2-}	7.2
$Co(NH_3)_6^{3+}$	10.2			F^-	5.50		

How **fast** does an ion move in the electric field?

$u_\infty \ in \ \frac{\mathrm{mm}^2}{\mathrm{V\,s}}$

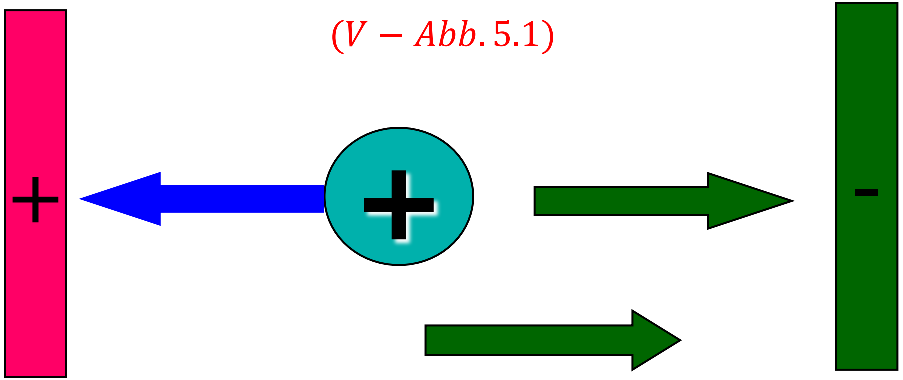
$Na^{+}: 0.052$

$Cl^{-}: 0.079$

$$F_{Stokes} = 6\,\eta\,\pi\,r\,\mathrm{v} \quad (V-5.2)$$

$$F_{el} = z\,e\,E \quad (V-5.1)$$

$(V - Abb. 5.1)$



$$\mathrm{v}_{+} = \frac{z_{+}\,e}{6\,\eta\,\pi\,r} E \quad (V-5.3)$$

$$\mathrm{v}_{+} = u_{+}\,E$$

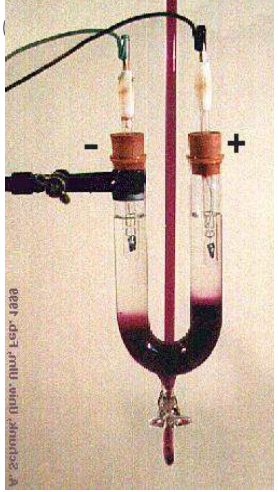
$$u_{+} = \frac{\lambda_{+}}{F} \quad (V-5.5)$$

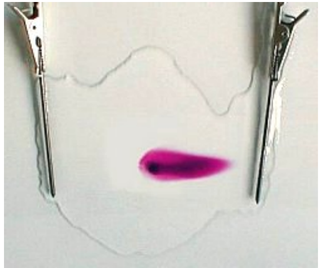
#ElectricField

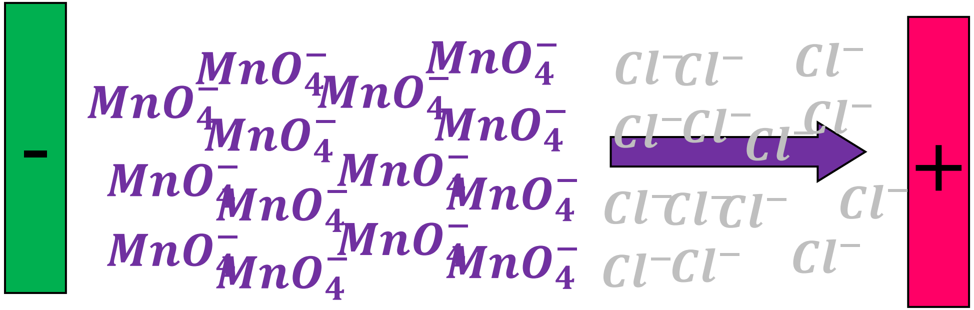
#DriftVelocity

#IonicMobility

How can we measure **velocity, mobility** and **radius** of an ion?







$$\mathbf{v}_- = \frac{\Delta x}{\Delta t}$$

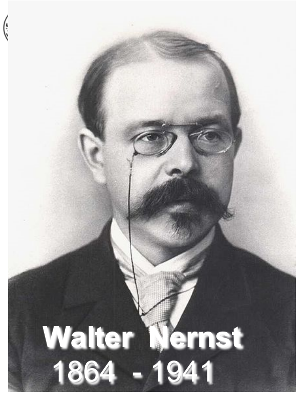
$$u_- = \frac{v_-}{E}$$

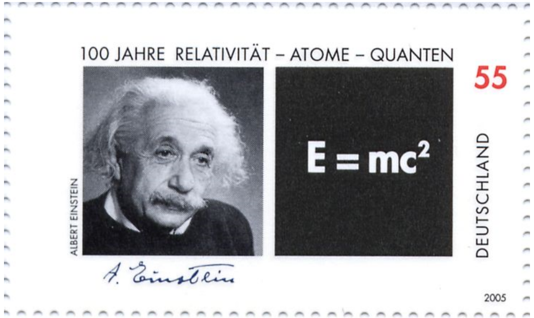
$$\lambda_- = u_- F$$

$(V - 5.5)$

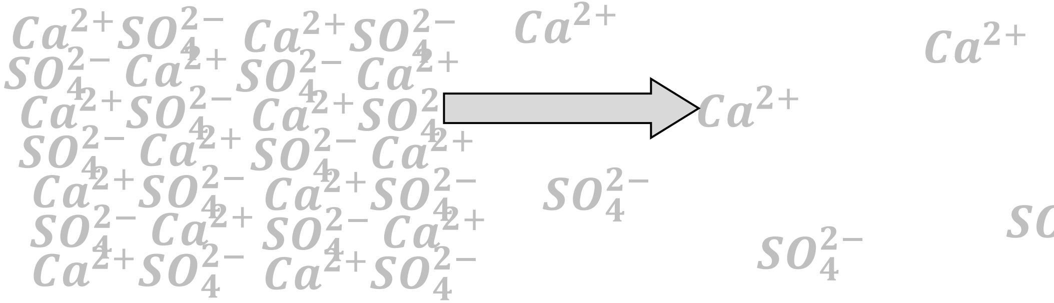
$$r_- = \frac{z_- \, e}{6\,\eta\,\pi\, u_-}$$

#Electrophoresis





How fast do ions **diffuse**?



$$\lambda_+ = \frac{z_+^2 F^2}{RT} D_+ \qquad \lambda_- = \frac{z_-^2 F^2}{RT} D_-$$

$(V-5.9)$

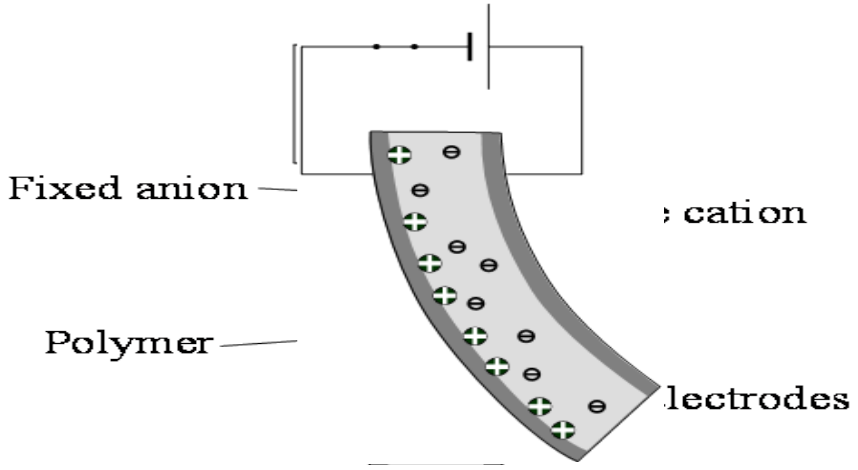
$$\bar{D} = \frac{2\, D_+\, D_-}{D_+ + D_-}$$

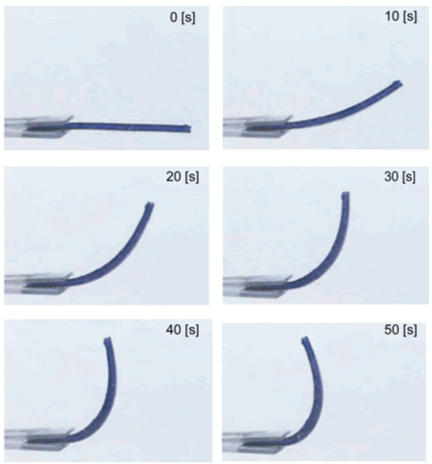
$(V-5.10)$

#NernstEinsteinEquation

Ion	H^+	Li^+	K^+	Ca^{2+}	Cu^{2+}	OH^-	Cl-	NO_3^-	SO_4^{2-}	Acetat
u / 10^{-4} cm²/(Vs)	36,23	4,01	7,62	6,17	5,55	20,64	7,91	7,4	8,29	4,34
λ_∞ / cm²/(Ω mol)	349,7	38,7	73,5	59,5	53,5	197,0	76,4	71,5	80,0	40,9

How do ions move in **polymer electrolytes**?





#ElectroactivePolymer

λ_∞ in $\frac{\mathrm{mS\ m^2}}{\mathrm{mol}}$

H^+: 35.0

$Acetat^-$: 4.09

Na^+: 5.01

Cl^-: 7.63

True or False?

A: Anions and cations always migrate at the same speed

B: Anions always migrate faster than cations

C: Protons are the ions which migrate fastest in water

D: Anions always migrate slower than cations

λ_∞ in $\frac{\text{mS m}^2}{\text{mol}}$

Li^+: 3.87

Cl^-: 7.63

Calculation **drift velocities** of anions and cations

3. Eine verdünnte Lithiumchlorid-Lösung befindet sich in einem elektrischen Feld der Feldstärke 1 V/cm. Berechnen Sie die Driftgeschwindigkeiten v_+ und v_- der Lithium- und der Chloridionen sowie deren Überführungszahlen t_+ und t_-.

Driftgeschwindigkeit eines Kaliumions im elektrischen Feld

Im letzten Kapitel haben wir die Leitfähigkeit einer Kaliumchloridlösung berechnet. Bei Anlegen einer Spannung von 10 V und einem Abstand der Elektroden von 1 m floss durch die Lösung ein Strom von 0,112 A.

Die Feldstärke im Inneren des Elektrolyten können wir mithilfe der einfachen Gleichung eines Plattenkondensators ermitteln:

$$E = \frac{U}{d} = \frac{10\,\text{V}}{1\,\text{m}} = 10\frac{\text{V}}{\text{m}} \quad (5.6)$$

Die Beweglichkeit des Kaliumions berechnen wir aus dessen Leitfähigkeit. Wir verwenden den Tabellenwert der Grenzleitfähigkeit für unendliche Verdünnung:

$$u_{K^+} = \frac{\lambda_{K^+}}{F} = \frac{0{,}00735\frac{\text{Sm}^2}{\text{mol}}}{96485\frac{\text{As}}{\text{mol}}} = 7{,}62\cdot 10^{-8}\frac{\text{m}^2}{\text{V}\cdot\text{s}} \quad (5.7)$$

Damit ergibt sich für die Driftgeschwindigkeit des Kaliumions:

$$v_{K^+} = u_{K^+}\cdot 7{,}62\cdot 10^{-8}\frac{\text{m}^2}{\text{V}\cdot\text{s}}\cdot 10\frac{\text{V}}{\text{s}} = 7{,}62\cdot 10^{-7}\frac{\text{m}}{\text{s}} = 0{,}27\frac{\text{mm}}{\text{h}} \quad (5.8)$$

Die Geschwindigkeiten der Ionen sind – mit makroskopischen Maßstäben gemessen – sehr gering. Mikroskopisch bedeuten 0,27 mm/h jedoch, dass sich das Ion pro Sekunde an über 1000 Wassermolekülen vorbei bewegt.

PCÜ45 Driftgeschwindigkeit von Ionen im elektrischen Feld - Ionen wandern im Schneckentempo

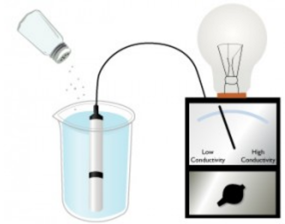
Physikalische Chemie by SciFox

How does **adding a small salt crystal** change the conductivity of water?

λ_∞ *in* $\frac{\mathrm{mS\ m^2}}{\mathrm{mol}}$

Na^+: 5.01

Cl^-: 7.63



$$\frac{(0.2\ mg)}{58.4\,\frac{g}{mol}} = 3.4\ \mu mol$$

$$NaCl\,(s) \rightarrow Na^+(aq) + Cl^-(aq)$$

$$\Lambda = 12.6\ \frac{\mathrm{mS}\ m^2}{mol}$$

$$\kappa = \Lambda c = 12.6\ \frac{mS\ m^2}{mol}\ 3.4\ \frac{mmol}{m^3} = 43\ \frac{\mu S}{m}$$

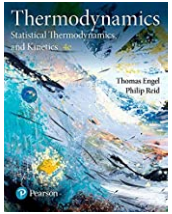
$$= 0.43\ \frac{\mu S}{cm}$$

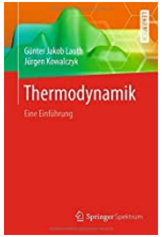
$Purest\ \ water: 0.05\ \frac{\mu S}{cm}$

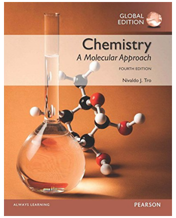
$Tap\ water: 500\ \frac{\mu S}{cm}$

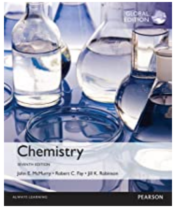
$Sea\ water: 50\ \frac{mS}{cm}$

REFERENCES:









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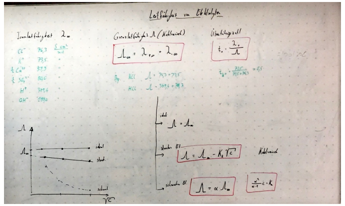
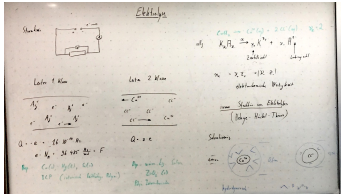
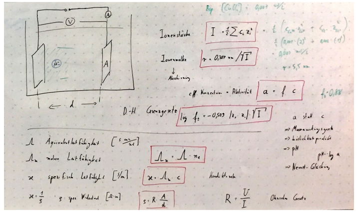
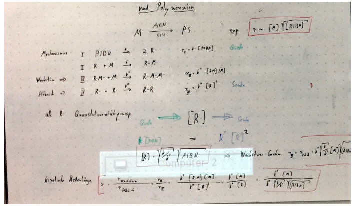






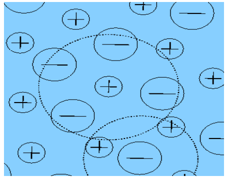


PHYSICAL CHEMISTRY BASICS

PART 11: ELECTROLYTIC CONDUCTIVITY

$$\propto$$

$$K_{\nu_+} A_{\nu_-} \rightarrow \nu_+ \; K^{z+} + \nu_- \; A^{z-}$$



$$n_e = \nu^+ z^+ = |\nu^- z^-|$$

$$I = \frac{1}{2} \sum_i z_i^2 \cdot c_i \qquad \log f_\pm = 0.509 \, z_+ \, z_- \sqrt{\frac{I}{\mathrm{mol/L}}}$$

$$\kappa = \frac{1}{R} \frac{l}{A} \qquad \Lambda = \frac{\kappa}{c \, n_e}$$

$$\Lambda_\infty = \lambda_\infty^+ + \lambda_\infty^-$$

$$t_+ = \frac{\lambda_+}{\Lambda}$$